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# Cosmic strings with interacting hot dark matter

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**ABSTRACT**

We compute the linear power spectrum of cosmic string sedeed fluctuations in the context of neutrinos with a strong self-interaction and show that it is very similar to that obtained in the context of 'normal' neutrinos. We compare our results with observational data and show that for any value of the cosmological parameters $h$ and $\Omega_0$ the interacting hot dark matter power spectrum requires a large scale dependent biasing parameter.

**Key words:**
large-scale structure of the Universe – dark matter – galaxies: clustering

## 1 INTRODUCTION

Observations on all scales of cosmological interest show that most of the matter in the universe is in some form of unseen matter. On the other hand, nucleosynthesis constraints seem to imply that most of this dark matter is non baryonic. Gravitinos, axions and neutrinos for example, are only a few among the many candidates to constitute the dark matter.

In the standard scenario it is usually assumed that during the important epochs for structure formation, the dark matter particles candidates interact only gravitationally. However one could assume that this is not the case. In particular it is possible that light neutrino-like particles could have some other kind of self-interaction during the low energy epochs when structure formation takes place (see, for instance, Raffelt and Silk 1987; Carlson, Machacek and Hall 1992; Gradwohl and Frieman 1992; Machacek 1994; Laix, Scherrer and Schaefer 1995).

In recent work Atrio-Barandela and Davidson (1997), have considered a light ($\sim 30\,\mathrm{eV}$) self-interacting neutrino-like particle and discussed the possibility that the dark matter in the universe could be constituted by this kind of particles. They determined the linear power spectrum of density fluctuations generated by the present time in the context of primordial gaussian fluctuations and concluded that galaxy sized density perturbations could survive.

In what concerns the estimates for the bounds on the neutrino-neutrino cross section very little is known. Within the framework of the Standard Model of Particle Physics, no experimental bounds have been set, and the best known lower limit on the mean free path of the neutrinos is based on observations of the supernova $SN\,1987\,A$. In fact, the neutrinos produced by $SN\,1987\,A$ in the Large Magellanic Cloud, reached the earth after crossing the relic neutrinos of the cosmic background. Since the number of detected neutrinos agrees with the theoretical expected number from supernova models, the present mean free path of the neutrinos $\lambda_0$, is not shorter than the distance to $SN\,1987\,A$, which is of the order of $2 \times 10^{23}$ cm; for a complete analysis, see Kolb and Turner, 1987; (see also Nussinov and Roncadelli, 1983). In this article we will assume that the co-moving value of $\lambda$ is much smaller than any scale of cosmological interest during the time where most of the structures we observe in the universe today are generated. In this case one can treat the neutrino component with the hydrodynamic or fluid approximation.

Here we extend the work of Barandela and Davidson (1997) to the case of non-gaussian density perturbations induced by cosmic strings. Cosmic strings, along with other topological defects, may be produced as a result of symmetry breaking phase transitions in the early universe (Kibble 1976). Although some topological defects are ruled out by observations, others like cosmic strings could be the seeds which generated the large scale structures we observe in the universe today. They also have other interesting cosmological consequences (see for example Vilenkin 1985; Colombi 1993; Vilenkin and Shellard 1994; Avelino 1996). Among these there is a non-gaussian signature in the microwave background (on small scales), the production of double images and a primordial background of gravitational waves or other radiation.

The outline of this paper is as follows. In section 2 we start by describing the formalism we employ in order to study the growth of density fluctuations in the context of neutrinos with a strong self-interaction. We introduce com-

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pensation in section 3 and in section 4 we describe the semi-analitic model we employ in order to compute the power spectrum of cosmic string seeded density fluctuations. In section 5 we describe and discuss the results. We conclude in section 6.

In this article employ fundamental units with $\hbar = c = k_B = 1$. We consider a self-interacting neutrino with mass $m_\nu = 93\Omega_0 h^2$ such that $\Omega_0 = \Omega_\nu$.

## 2 SUBSEQUENT PERTURBATIONS

In a flat FRW universe with no cosmological constant and containing both CDM and radiation fluids, the scale factor $a$ may be written in terms of the conformal time $\eta$ as

$$a(\eta) = a_{eq}\left[2(\sqrt{2}-1)\frac{\eta}{\eta_{eq}} + (3-2\sqrt{2})\frac{\eta^2}{\eta_{eq}^2}\right], \tag{1}$$

where $a_{eq}$ and $\eta_{eq}$ are respectively the expansion factor and conformal time at matter-radiation equality. If the dark matter is hot but the transition from the relativistic to the non-relativistic regime occurs deep in the radiation era eqn. (1) is still a good approximation.

The evolution of radiation and non-relativistic interacting hot dark matter (IHDM) density fluctuations in the synchronous gauge is given by

$$\ddot{\delta}_h + \frac{\dot{a}}{a}\dot{\delta}_h - \frac{3}{2}\left(\frac{\dot{a}}{a}\right)^2(\Omega_h\delta_h + 2\Omega_r\delta_r) - c_s^2\nabla^2\delta_h = 4\pi(\Theta_{00}+\Theta_{ii}), \tag{2}$$

$$\ddot{\delta}_r - \frac{1}{3}\nabla^2\delta_r - \frac{4}{3}\ddot{\delta}_h = 0, \tag{3}$$

where $\Theta_{\alpha\beta}$ is the energy-momentum tensor of the external source, $c_s$ is the adiabatic sound speed of the interacting hot dark matter and $\Omega_h$ and $\Omega_r$ express the densities in radiation and IHDM as fractions of the critical density.

We are implicitly assuming that the strength of the neutrino-neutrino coupling is large enough for it to be a good approximation to treat the neutrino component as a perfect fluid. We also assume the radiation behaves as a fluid; this is certainly valid before recombination for the length scales of interest, well above the radiation damping scale. Although this is not true after recombination, for $\Omega_0 h^2$ not too small the radiation no longer dominates the dynamics after last scattering. Consequently, we expect that even then equation (3) remains a valid approximation.

The solution to the system of equations (2,3) with initial conditions $\delta_h = \delta_r = \dot{\delta}_h = \dot{\delta}_r = 0$ can be written in terms of Green functions as

$$\delta^S_{h,r}(\mathbf{x},\eta) = 4\pi\int_{\eta_i}^{\eta} d\eta' \int d^3x' \mathcal{G}^{h,r}(X;\eta,\eta')\Theta_+(\mathbf{x}',\eta'), \tag{4}$$

$$\mathcal{G}^{h,r}(X;\eta,\eta') = \frac{1}{2\pi^2}\int_0^\infty \widetilde{\mathcal{G}}^{h,r}(k;\eta,\eta')\frac{\sin kX}{kX}k^2 dk. \tag{5}$$

Here, $\Theta_+ \equiv \Theta_{00} + \Theta_{ii}$ and $X = |\mathbf{x}-\mathbf{x}'|$. The upper index 'S' indicates that these are the 'subsequent' fluctuations, according to the notation of Veeraraghavan & Stebbins (1990), to be distinguished from the 'initial' fluctuations. In Fourier space, the Green function $\widetilde{\mathcal{G}}^{h,r}$ obey the same equations as $\delta_{h,r}$:

[t]

**Figure 1.** Plot of $T$, $\mu_p$ (in units of $m$), and $c_s$ as a function of the scale factor $a$ normalized to unity at $\eta_{\rm eq}$.

$$\ddot{\widetilde{\mathcal{G}}}{}^h + \frac{\dot{a}}{a}\dot{\widetilde{\mathcal{G}}}{}^h - \frac{3}{2}\left(\frac{\dot{a}}{a}\right)^2(\Omega_h\widetilde{\mathcal{G}}^h + 2\Omega_r\widetilde{\mathcal{G}}^r) + c_s^2k^2\widetilde{\mathcal{G}}^h = 0, \tag{6}$$

$$\ddot{\widetilde{\mathcal{G}}}{}^r - \frac{4}{3}\ddot{\widetilde{\mathcal{G}}}{}^h + \frac{1}{3}k^2\widetilde{\mathcal{G}}^r = 0. \tag{7}$$

The initial conditions at $\eta_i$ are $\dot{\widetilde{\mathcal{G}}}{}^h = 1$, $\dot{\widetilde{\mathcal{G}}}{}^r = 4/3$ and $\widetilde{\mathcal{G}}^h = \widetilde{\mathcal{G}}^r = 0$. Hence, given the evolution of the source stress-energy, it is possible to numerically compute the resulting density fluctuations.

We are mostly interested in the interacting IHDM inhomogeneities at late times in the matter era. In the limit $\eta \gg \eta_{eq}$, the Green functions are dominated by the growing mode, $\propto a/a_{eq}$. Hence, the function we would like to solve for is

$$T(k;\eta_i) = \lim_{\eta/\eta_{eq}\to\infty} \frac{a_{eq}}{a}\widetilde{\mathcal{G}}^h(k,\eta,\eta_i). \tag{8}$$

This function will be used later to construct the transfer function for the power spectrum of IHDM density fluctuations induced by cosmic strings.

### 2.1 The IHDM sound speed

The adiabatic sound speed of the self-interacting neutrinos $c_s$, is given by

$$c_s^2 = \left(\frac{\partial p}{\partial \rho}\right)_S, \tag{9}$$

where $p$ stands for pressure, $\rho$ is the mass density of the neutrinos and the subscript $S$ on the right hand side means that the entropy per particle $S$, is constant.

The neutrinos have Fermi-Dirac distribution function. Therefore, the neutrino number density $n$, mass density $\rho$ and pressure $p$ are given respectively by:

$$n = A\int_m^\infty E\,\left(E^2-m^2\right)^{1/2}\frac{1}{\exp\left[(E-\mu_p)/T\right]+1}dE, \tag{10}$$

$$\rho = A \int_m^\infty E^2 \left(E^2 - m^2\right)^{1/2} \frac{1}{\exp\left[(E-\mu_p)/T\right]+1}\, dE, \quad (11)$$

$$p = \frac{A}{3} \int_m^\infty \left(E^2 - m^2\right)^{3/2} \frac{1}{\exp\left[(E-\mu_p)/T\right]+1}\, dE, \quad (12)$$

where $E = \left(\mathbf{p}^2 + m^2\right)^{1/2}$ is the energy, $\mathbf{p}$ is the momentum, $m$ is the mass, $T$ is the temperature, $\mu_p \equiv \mu_p(T)$ is the chemical potential, $A = g/(2\pi)^2$, and $g$ is the spin degeneracy factor of the neutrinos. Using the constraint

$$S = \frac{1}{T}\left(\frac{p+\rho}{n} - \mu_p\right) = \frac{7\pi^4}{135\zeta(3)}, \quad (13)$$

where $S$ is the entropy per neutrino, we numerically calculated $n$, $\rho$, $p$ and $\mu_p$ as functions of $T$. We then obtained the adiabatic sound speed $c_s^2 = (\partial p/\partial \rho)_S$, and fit the obtained result by:

$$c_s(a) = \frac{\sqrt{3}}{3}\left[1 + \left(\frac{a}{\alpha}\right)^2\right]^{-1/2}, \quad (14)$$

where $\alpha = 0.19$ is the best fit parameter. In Fig. 1 we represent the numerical plots of $T$ and $\mu_p$ (in units of $m$), and $c_s$, as functions of the scale factor $a$, normalized to unity at the epoch of equality between matter and radiation.

## 3 COMPENSATION

The linear perturbations induced by cosmic strings, are the sum of initial and subsequent perturbations:

$$\begin{aligned}\delta_h(k;\eta) &= \delta_h^I(k;\eta) + \delta_h^S(k;\eta) \\ &= 4\pi(1+z_{eq}) \int_{\eta_i}^{\eta} d\eta'\, T(k;\eta')\widetilde{\Theta}_+(k;\eta'). \quad (15)\end{aligned}$$

The transfer function for the subsequent perturbations, those generated actively, was obtained in the previous section. To include compensation for the initial perturbations, $\delta_h^I$, we make the substitution:

$$T(k;\eta) \rightarrow \left(1 + (k_c/k)^2\right)^{-1} T(k;\eta), \quad (16)$$

where $k_c$ is a long-wavelength cut-off at the compensation scale. This results from the fact that local physical processes cannot produce perturbations on scales much larger than the horizon size. Consequently, the power spectrum of density perturbations is bounded by a $k^4$ spectrum on large scales ($k << k_c$), assuming that background fluctuations are uncorrelated for points which have never been in causal contact. In the I-model of strings, which most resembles the Bennett-Bouchet (1990) and Allen-Shellard simulations (1990), $k_c = 2\pi/\eta$.

## 4 POWER SPECTRUM

The analytic expression of Albrecht and Stebbins (1992) for the power spectrum of density perturbations induced by cosmic strings in a $\Omega = 1$ FRW universe with no cosmological constant is

$$P(k) = 16\pi^2(1+z_{eq})^2(G\mu)^2$$

[t]

**Figure 2.** Comparison between the CMB-normalized linear power spectrum of cosmic string seeded fluactuations with CDM (dot-dashed line), IHDM (solid line) and HDM (doted line) for $\Omega_0 = 1$ and $h = 1$.

$$\times \int_{\eta_i}^{\eta_o} d\eta' \mathcal{F}[k\xi(\eta')/a(\eta')] |T(k,\eta')|^2, \quad (17)$$

where the "$\rho + 3p$" part of the string stress energy tensor, is modeled by $\mathcal{F}$ given by

$$\mathcal{F}[k\xi/a] = \frac{2}{\pi^2}\beta^2\Sigma\frac{\chi^2}{\xi^2}\left(1 + 2(k\chi/a)^2\right)^{-1}. \quad (18)$$

In these equations, $\eta_0$ is the conformal time today, $\eta_i$ is the conformal time when the string network was formed, $\chi$ is the typical curvature scale of the wakes,

$$\xi \equiv (\rho_\infty/\mu)^{\frac{1}{2}}, \quad \beta \equiv \langle v^2\rangle^{1/2}, \quad (19)$$

$$\Sigma \equiv \frac{\mu_r}{\mu}\gamma_b\beta_b + \frac{1}{2\gamma_b\beta_b}\left(\frac{\mu_r^2 - \mu^2}{\mu_r \mu}\right), \quad (20)$$

where $\rho_\infty$ is the energy density in long strings, $\mu$ is the string mass per unit length, $v$ is the microscopic velocity of the string, $\beta_b$ is the macroscopic bulk velocity of the string, $\gamma_b = (1-\beta_b^2)^{\frac{1}{2}}$, and $\mu_r$ is the renormalized mass-per-unit-length. We note that $\Sigma\beta^2 \sim 1$ and $\chi/a \sim 2\xi/a \sim \eta/3$ both in the matter and radiation eras (Bennett and Bouchet 1990; Allen and Shellard 1990) and consequently the structure function $\mathcal{F}$ is always well approximated by

$$\mathcal{F}[k\eta] = \frac{8}{\pi^2}\left[1 + \frac{2}{9}(k\eta)^2\right]^{-1}. \quad (21)$$

We substitute this in equation (17) in order to calculate the IHDM power spectrum.

## 5 RESULTS

In Fig. 2 we compare the power spectrum of the perturbations generated by cosmic strings for both CDM (dot-dashed line), HDM (doted line) and IHDM (solid line) for $h = 1$ and

[t]

**Figure 3.** Comparison between the CMB-normalized linear power spectrum of cosmic string seeded fluactuations with IHDM (solid line) for $h = 0.7$ and $\Omega_0 = 1.0, 0.4, 0.2$. For $\Omega < 1$ the reconstructed linear power spectrum has been rescaled by $\Omega^{-0.3}$ (Peacock and Dodds, 1994).

$\Omega_0 = 1$. The curves are normalized to the COBE-DMR observations (Allen el al., 1997). We can see that the HDM and IHDM power spectra are very similar, the IHDM power spectrum having a slightly larger amplitude on small scales. The absence of oscillations reflects the incoherent nature of string perturbations which is implicit in the way we add the perturbations generated by the cosmic strings at different times to calculate the power spectrum. Hence, it seems that the IHDM cosmic string scenario suffers from similar problems as the standard HDM scenario for cosmic strings. We shall discuss these problems next when we investigate other choices for the cosmological parameters.

### 5.1 Open models

The generalization of these results for open models of structure formation can be made using the same technics employed by Avelino, Martins and Caldwell (1997). In Fig. 3 we compare the power spectrum of the perturbations generated by cosmic strings in the context of IHDM with the Peacock and Dodds (1994) linear power spectrum reconstrucion inferred from various galaxy surveys.

The IHDM power spectrum is again normalized to the COBE-DMR observations (Allen el al., 1997). We see that in an open universe it is possible to obtain a better fit to the shape of the Peacock and Dodds linear power spectrum on large scales. However, in this case perturbations are damped on too large scales which is in conflict with the observational data. In a flat universe with a cosmological constant the power spectrum requires a slightly lower biasing than for an open universe with the same matter density (Avelino, Caldwell and Martins, 1997). However, because the shape of the power spectrum density fluctuations induced by cosmic strings remains the same this model is also in conflict with observations.

## 6 CONCLUSIONS

In this paper we calculated the power spectrum of density fluctuations induced by cosmic strings in the contex of neutrinos with strong self-interactions. We concluded that because gravitational instability can only be established on scales smaller than the Jeans scale, small scale power is removed relative to the CDM case. In opposition to the case studied by Barandela and Davidson (1997), we note the absence of oscillations, which is due to the incoherent nature of the cosmic strings source. In fact, the results obtained for HDM and IHDM are very similar, the IHDM spectrum having a slightly larger amplitude on small scales. The generalization of these results shows that in an open universe or a flat universe with a cosmological constant it is possible to decrease the biasing on large scales by a small factor. However, in this case perturbations are damped on too large scales. Hence, we conclude that it seems difficult to reconcile these results with observations unless there is some physical mechanism which is capable of generating a large, scale dependent biasing.

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**Fig. 1**

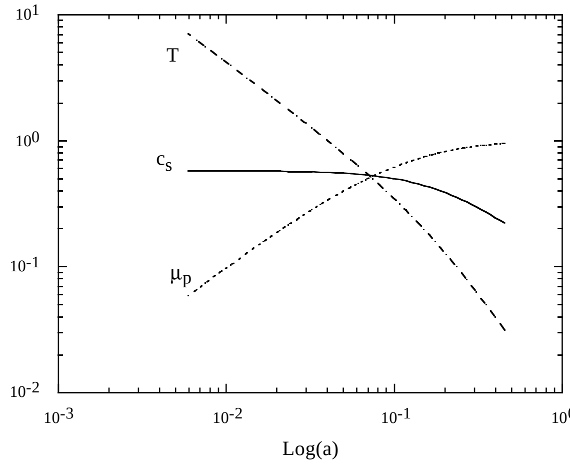

**Fig. 2**

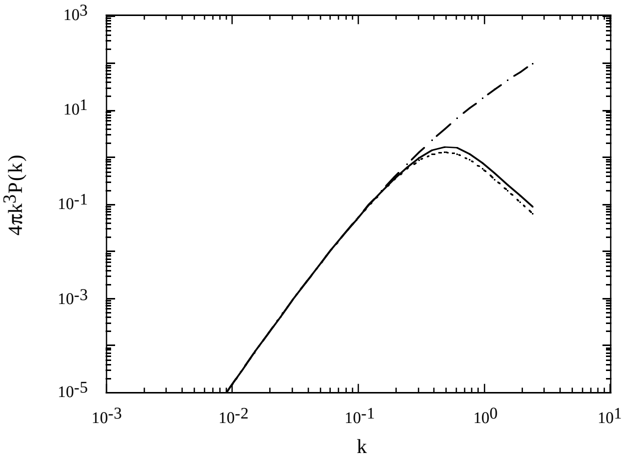

**Fig. 3**